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ON THE EVALUATION OF THE PARAMETER σ_0 IN CHAPMAN'S FORMULA FOR DETERMINING THE IONIC DENSITY OF THE E LAYER

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ABSTRACT. In our previous paper it has been mentioned that the discrepancy of the theoretical values of ionic densities from experimental ones for E layer is due mainly to the assumption of taking σ_0 constant during day and night. In the present paper the authors give the values of σ_0 which seem to give the best agreement possible.

In a previous paper (Sengupta and Dutt, 1941) it was shown by the authors that in the early part of the night of winter months of 1940-41 radio waves of frequency 810 kc/s transmitted from the Calcutta Station (Lat. 22°34'N, Long. 88°55'E) reached Patna (Lat. 25°30'N, Long. 85°15'E) after reflection from the E layer while in the same period of night of summer months of 1941 those waves penetrated the E layer. It was also pointed out that our results were in agreement with those obtained at Watheroo (Lat. 30° 19'S, Long. 115° 53'E) which has almost the same latitude as Patna but in the opposite hemisphere. This type of result could not be predicted from Chapman's (1931) formula with a constant value of the parameter σ_0 in the formula throughout day and night and for all seasons. In the present work an attempt has been made to find suitable day-time and night-time values of σ_0 for summer and winter months.

Curves showing the variation of ion density with time (ν, ϕ) for June and December of Watheroo, are given in Figs. 1 and 2. Tables I and II give the values of the critical frequencies (f_c) for E layer at Watheroo observed by Parkinson and Prior (1939, 1940, 1941) by the modern automatic multi frequency equipment for June of years 1938 to 1941 and for December of years 1938 to 1940 respectively. Square values of the critical frequencies are calculated and finally the squares reduced to fractions by a suitable choice of a factor such that the midday value (obtained from the graph of hours and squares of critical frequency) corresponded to the theoretical value of ν where

$\nu = n/n_0$; n = ion-density at any time.

n_0 = the maximum steady value of n if the rate of production of ions were to remain constant at its maximum possible value.

All the critical frequencies f_c in Mc/s are for the E layer at Watheroo (lat. 30° 19'S, Long. 115° 53'E).

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TABLE I

(Winter)

120°E mean time Hours	June, 1938		June, 1939		June, 1940		June, 1941		Mean* ν
	f_c	ν	f_c	ν	f_c	ν	f_c	ν	
12	3.43	0.773	3.51	0.773	3.37	0.773	3.04	0.773	0.773
13	3.36	0.742	3.44	0.742	3.30	0.741	3.03	0.768	0.748
14	3.25	0.603	3.37	0.713	3.07	0.641	2.96	0.733	0.6[illegible]5
15	3.04	0.607	3.05	0.584	2.86	0.557	2.74	0.628	0.594
16	2.69	0.475	2.61	0.427	2.47	0.415	2.33	0.4[illegible]4	0.443
17	2.05	0.276	1.91	0.229	1.85	0.233	1.65	0.228	0.242
18			1.17	0.086					
19			1.00	0.063					
20			.94	.055					
21			.81	.041					
22			.73	.033					
23			.75	.035					
00			.70	.031					
01			.69	.030					
02			.67	.028					
03			.66	.027					
04			.68	.029					
05			.79	.039					
06			.85	.045	0.78	0.041			0.043
07	1.65	0.179	1.74	.190	1.61	.176	1.52	0.193	.184
08	2.57	.433	2.55	.408	2.50	.425	2.28	.435	.425
09	2.93	.564	3.02	.572	2.90	.572	2.57	.552	.565
10	3.21	.676	3.27	.672	3.11	.658	2.90	.704	.677
11	3.34	.732	3.41	.730	3.25	.719	3.00	.753	.734

*Hourly critical frequencies for the whole night are available for June, 1939 and December, 1938 only

Prof Chapman's theory (1931) of ionization of the upper atmosphere has been applied to construct curves for Watheroo showing the variation of ionic density with time (ν, ϕ). (The notation is according to Chapman's paper). In actual construction the simple approximate method of Millington (1932) together with his envelope correction (1935) has been made use of for solving the fundamental differential equation and obtaining the upper limit of theoretical ν values. Attempts were made to find the best fit with the experimental values by taking different constant values for both day and night of parameter σ_0 such as 0.01, 0.02, 0.03, etc. and the results for December and June are shown in Figs. 1 and 2. Here the average declination of the sun for the respective months has been considered and for greater accuracy points were obtained at close intervals by making $\phi_2 - \phi_1 = 0.1$ for a few hours around noon and $\phi_2 - \phi_1 = 0.05$ for the hours near sunset and sunrise. These results indicate that by taking σ_0 constant for day and night there is no good fit. It may be noted that for a few hours round about noon, curves drawn with different values of σ_0 practically overlap. The effect of alteration in the value of σ_0 is most pronounced during an hour or so before sunset. The value of σ_0 corresponding to the best agreement between the theoretical

TABLE II

(Summer)

120°E mean time Hours	December, 1938		December, 1939		December, 1940		Mean ν
	f_c	ν	f_c	ν	f_c	ν	
12	4.02	0.996	3.74	0.996	3.59	...	0.996
13	3.96	.967	3.74	.996	3.64	0.974	.979
14	3.83	.904	3.59	.918	3.58	.943	.922
15	3.80	.890	3.61	.877	3.45	.875	.881
16	3.50	.755	3.26	.757	3.21	.757	.756
17	3.10	.592	2.96	.599	2.85	.597	.596
18	2.44	.367	2.23	.354	2.22	.362	.361
19	1.46	.131	1.39	.137	1.45	.154	.141
20	.75	.0345					
21	.63	.0247					
22	.58	.0210					
23	.53	.0173					
00	.54	.0179					
01	.53	.0173					
02	.56	.0191					
03	.57	.0197					
04	.60	0.024	0.65	0.030			0.026
05	1.46	.131	1.44	.147	1.43	0.150	.143
06	2.45	.370	2.29	.373	2.19	.353	.365
07	3.06	.577	2.82	.566	2.69	.532	.558
08	3.47	.742	3.20	.729	3.12	.716	.729
09	3.79	.885	3.46	.852	3.42	.860	.866
10	3.95	.962	3.66	.923	3.55	.927	.937
11	4.01	.991	3.70	.975	3.67	.991	.986

and experimental curves during this period, was, therefore, taken as the value giving a good fit. The value of $\sigma_0 = 0.01$ no doubt agrees closely with the summer month of December of Watheroo and 0.03 for winter month of June for day-time only; but during night-time the agreement fails.

Now, according to the theory there should be continuous decay of ionic density during the night. The proper values of σ_0, therefore, for the night were obtained directly from the experimental observations of continuous decay of the ionic density during night. It may be noted, however, that for the evaluation of the above, observations were confined to early part of night only to avoid complications due to replenishing of the ionization which has been observed by Mitra (1938) and by Best, Farmer and Ratcliffe (1938) in the later part of night about 4 hours before sunrise. The values of night-time σ_0 that were obtained directly are given in Table III and are found to be different for different seasons. They are also found to differ from day-time values ($\sigma_0 = 0.01$ for December and $\sigma_0 = 0.03$ for June) giving the best fit.

TABLE III

Summer		σ_0	Winter	σ_0
December, 1938	...	0.029	June, 1939	0.059
January, 1939	...	0.023	July, 1939	0.064

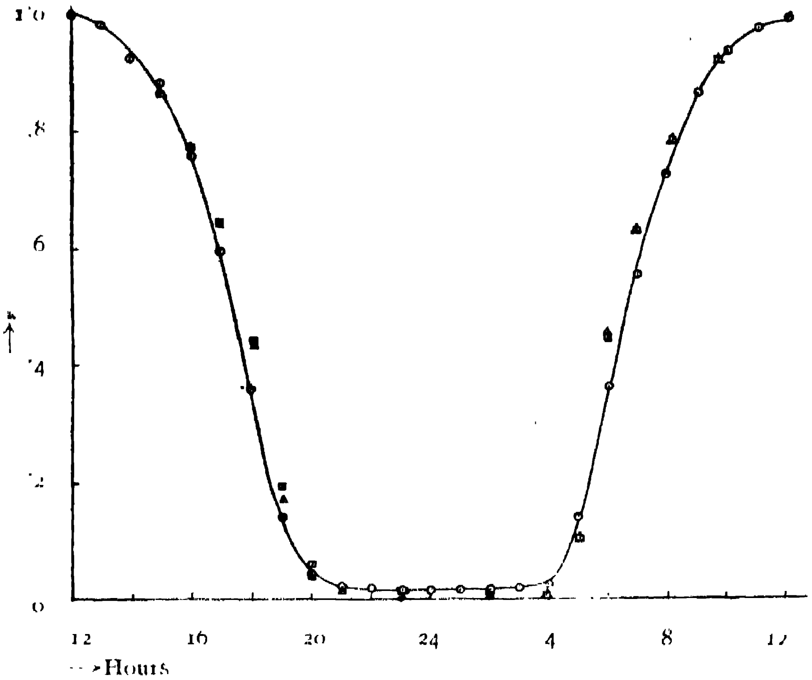

FIG. 1

December Curves

⊙ Experimental curve (mean for years 1938-40

□ ..Points of theoretical curve with σ_0=0.02

△...Points of theoretical curve with σ_0=0.01

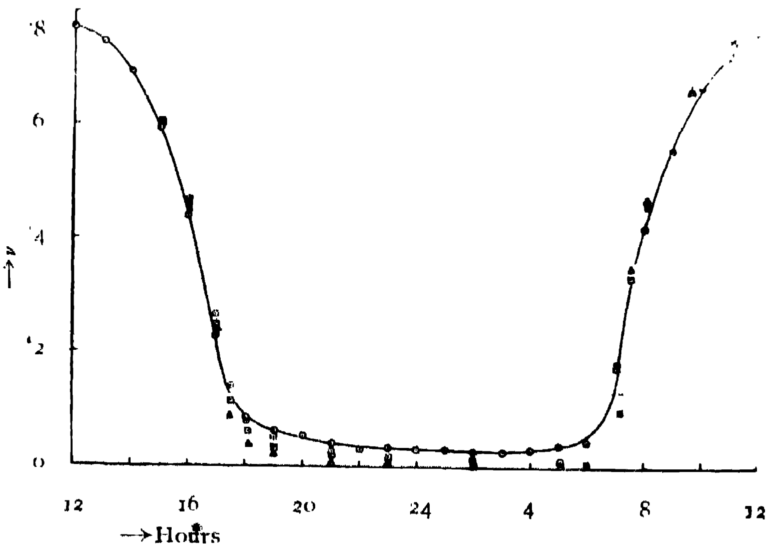

FIG. 2

June Curves

⊙...Experimental curves (mean for years 1938-41)

∵...Points of theoretical curve with σ_0=.03

□ ..Points of theoretical curve with σ_0= 02

△...Points of theoretical curve with σ_0=.01

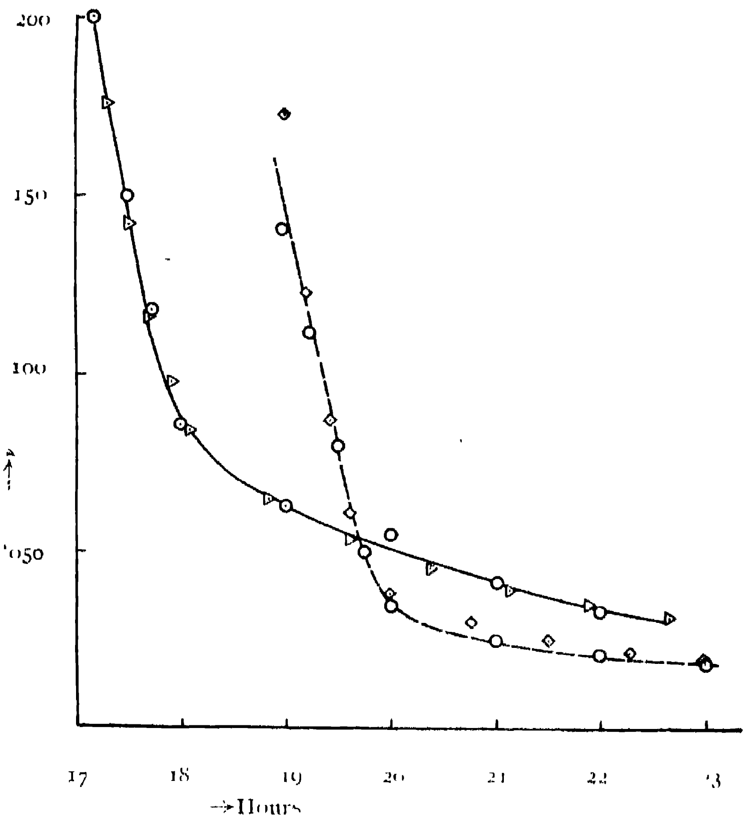

FIG. 3

Δ, Δ ...Theoretical curve

⊙ ..Experimental curve

June, 39 curve (full line)
$\sigma_0 = 0.03$ (day time)
$\sigma_0 = 0.059$ (night-time)

December, 38 curve (broken line)
$\sigma_0 = 0.01$ (day-time)
$\sigma_0 = 0.029$ (night-time)

To obtain the best fit, therefore, it is necessary to take different values of σ_0 for seasons as well as for day and night. The theoretical curves of Fig. 3 are drawn with $\sigma_0 = 0.01$ (day-time), $\sigma_0 = 0.029$ (night-time) for December (summer) and with $\sigma_0 = 0.03$ (day-time), $\sigma_0 = 0.059$ (night-time) for June (winter) in which the experimental data are also plotted for comparison ; the agreement comes out to be very close both for day and night and for seasons also.

Summer and winter (ν, φ) curves derived from Chapman's formula with a constant values of σ_0 for day and night throughout the year, will not intersect each other as the rate of ionization at any hour in summer is greater than that in winter for the same hour. But with different values of σ_0 the theoretical curves (Fig. 3) do intersect and agree with the experimental observations. The net result of different values of σ_0 is that the night-time ionic density of E layer for winter is larger than that for summer, though the noon value for the latter season is much higher than that for the winter.

That σ_0 has different values for night and day has been observed by Best, Farmer and Ratcliffe (*loc. cit.*). They found that for both summer and winter the night-time value of σ_0 is 0.098 corresponding to recombination co-efficient $\alpha = 4 \times 10^{-9}$ and day-time value of σ_0 a little greater than 0.032 corresponding to $\alpha = 1.2 \times 10^{-8}$. That it is necessary to assume a greater rate of recombination during the day than during the night, is confirmed by some curves calculated by Wilkes (1939) from observations made at Washington (Lat. 38° 50′ N, Long. 77° 00′ W) on 31st August, 1932, when an eclipse of the sun took place, & by Kirby, Gilliland and Judson (1936). He points out that the day-time curves on eclipse day with $\sigma_0 = 0$, α very large or $\sigma_0 = 0.032$, fit in with the experimental curves better than the one for $\sigma_0 = 0.098$. Hulburt (1939) concludes that during day-time the available data favour $\sigma_0 = 0.010$ ($\alpha = 2 \times 10^{-8}$) but do not preclude an even lower value of σ_0, *i.e.*, a higher rate of recombination.

Another point to be noted is that the summer night time value of σ_0 for Watheroo is found to be approximately 3 times the day-time value of σ_0 while winter night-time value is twice that during day-time. The night-time and day-time values of σ_0 as observed by Best, Farmer and Ratcliffe are in the ratio of 3 : 1.

From the above it is clear that the values of σ_0 obtained from the data of Watheroo are quite probable in view of the determinations of Hulburt, and of Best, Farmer and Ratcliffe. It has been shown by us (Sengupta and Dutt, 1944, that the behaviour of the ionosphere in the Patna-Calcutta region (about 24° N latitude) has been experimentally found to be similar to that of Watheroo, so far as the ionic densities of summer and winter are concerned ; it, therefore, seems that the most probable values of σ_0 for latitudes 24° to 30° North or South, may be taken as given above.

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